CALCULATION OF A CONICAL ANODE RADIATOR

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Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 2, pp. 240–244, 1966

UDC 536.3

A method of determining the dimensions of a minimum-weight anode radiator of a five-element thermionic generator for certain cases of variation of the cross-sectional area of the radiator is described.

The anode radiator considered is a system of cones with apex angle $\pi/2$ fitted on single disk-shaped thermionic elements mounted on five faces of a cube—a solar energy receiver (Fig. 1). The initial information required for the determination of the dimensions of a radiator of minimum weight is the law of variation of the cross-sectional area of the radiator, the amount of heat removed from the anode, the temperature of the anode, and the dimensions of the anode. The following cases of variation of the cross-sectional area of the radiator are considered:

The cross-sectional area increases linearly from the base of the cone to the periphery, so that the thickness of the cone wall remains constant.

The cross-sectional area of the cone is constant along the generatrix.

The cross-sectional area decreases linearly by 40% from the base of the cone to the periphery.

The cross-sectional area decreases linearly by 80% from the base of the cone to the periphery.

In the case of a cone the temperature distribution over its surface will be axisymmetric. Hence, the heat balance equation for an element of the cone will have the form

$$d\left(\lambda F_x \frac{dT_x}{dx}\right) = \Pi_x \varepsilon (\sigma T_x^4 - q_{inc}),$$

where q_{inc} is the radiant flux density from the surface of the cone on the element considered. We transform the heat balance equation, using the ratios

$$\varphi = F_x/F_0, \quad \Psi = \Pi_x/\Pi_0, \quad \Theta = T_x/T_0, \quad t = x/L.$$

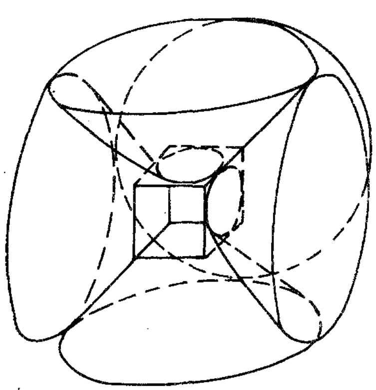

Fig. 1. Diagram of radiator.

After transformation we obtain

$$\frac{d(\varphi\, d\Theta/dt)}{dt} = C\Psi\Theta^4(1-\bar{q}_{inc}). \tag{1}$$

In this equation the conductivity parameter is

$$C = \varepsilon\sigma T_0^3 L^2 \Pi_0/\lambda F_0, \tag{2}$$

and the relative radiant flux density is

$$\bar{q}_{inc} = q_{inc}/\sigma T^4.$$

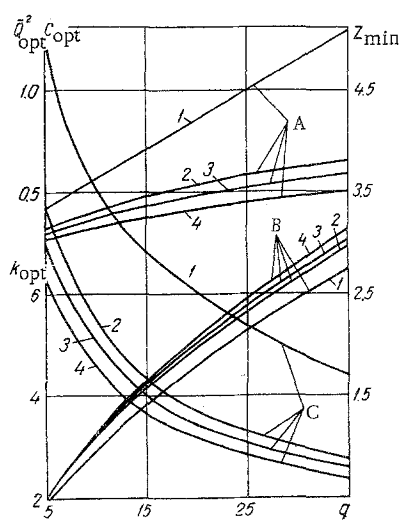

Fig. 2. Plots of $\bar{Q}^2_{opt}\, C_{opt}$ (A), k_{opt} (B), and Z_{min} (C) against degree of intensification of heat removal q: 1) cross-sectional area increases linearly from base of radiator to periphery so that wall thickness remains constant ($\varphi = 1 + kt$); 2) cross-sectional area of cone is constant along generatrix (φ = const); 3) cross-sectional area decreases linearly by 40% from base of radiator to periphery ($\varphi = 1 - 0.4t$); 4) cross-sectional area decreases linearly by 80% from base of radiator to periphery ($\varphi = 1 - 0.8t$).

We write the boundary conditions for Eq. (1), neglecting heat emission from the end of the cone

$$\Theta = 1 \text{ when } t = 0,$$

$$d\Theta/dt = 0 \text{ when } t = 1. \tag{3}$$

Function φ depends on the adopted law of variation of the cross-sectional area of the radiator. As indicated above, we consider various φ

$$\varphi = 1 + kt, \quad \varphi = \text{const},$$

$$\varphi = 1 - 0.4t, \quad \varphi = 1 - 0.8t, \tag{4}$$

where

$$k = L_0 \sin(\pi/4)/r_0.$$

For a cone

$$\Psi = 1 + kt. \tag{5}$$

Principal Results of Calculations

Parameters	Numercial values for φ equal to			
	$1+kt$	const	$1-0.4t$	$1-0.8t$
$\gamma_{a.r}$, N/kW	15.3	9.61	8.63	7.65
h_0, mm	2.64	3.61	4.0	4.56
L, cm	10.6	11.6	11.6	11.6

The radiant flux density depends on the position of the irradiated element of area along the generatrix of the cone and the temperature distribution over its surface. The density of the incident radiant flux is determined from the known integral relations for radiant heat transfer between bodies in a transparent medium [1, 2]

$$\bar{q}_{\text{inc}} = f\,[t,\ \Theta(t)]. \tag{6}$$

To solve Eq. (2) in conjunction with Eqs. (4)–(6) we must have two additional conditions. These conditions are embodied in the formulation of the problem and are: The radiator must remove a specified amount of heat Q_{rem} and must be of minimum weight. We write the first condition in the form

$$Q_{\text{rem}} = \varepsilon\sigma T_0^4 \Pi_0 L\bar{Q}. \tag{7}$$

In this equation the efficiency of the cone is

$$\bar{Q} = \int_0^1 \Theta^4 (1 - \bar{q}_{\text{inc}})\, dt. \tag{8}$$

We denote by q the degree of intensification of heat removal, equal to the ratio of the heat which must be removed from the anode to the heat which the anode surface can emit,

$$q = Q_{\text{rem}} / \varepsilon\sigma T_0^4 \pi r_0^2. \tag{9}$$

We will assume that the outer surface of the cones does not take part in heat removal, and hence

$$\Pi_0 = 2\pi r_0. \tag{10}$$

We transform equality (7), using the relations (4), (9), and (10). We obtain the first additional condition

$$k\bar{Q} = q \sin(\pi/4)/2. \tag{11}$$

We consider the condition imposed by the minimum weight requirement. For this purpose we write the expression for the specific weight $\gamma_{a.r}$ of the radiator in the form

$$\gamma_{a.r} = \frac{G_{a.r}}{Q_{\text{rem}}} = g\rho \frac{F_0}{\Pi_0} \frac{1}{\varepsilon\sigma T_0^4} \frac{\alpha}{\bar{Q}}, \tag{12}$$

where

$$\alpha = \int_0^1 \varphi\, dt.$$

To determine F_0/Π_0 we turn to equality (2)

$$F_0/\Pi_0 = \varepsilon\sigma T_0^3 L^2 / C\lambda.$$

We substitute F_0/Π_0 in Eq. (12). We now multiply the numerator on the right side of the obtained expression by the square of the left side of Eq. (7), and the denominator by the square of the right side. After multiplication we obtain

$$\gamma_{a.r} = g\rho \frac{(Q_{\text{rem}}/\Pi_0)^2}{\lambda\varepsilon^2\sigma^2 T_0^9} \frac{\alpha}{\bar{Q}^3 C}.$$

Using q and Π_0 we transform this expression

$$\gamma_{a.r} = g \frac{\rho}{\lambda} \frac{r_0^2}{4T_0} q^2 \frac{\alpha}{\bar{q}^3 C}. \tag{13}$$

We put

$$\alpha/\bar{Q}^3 C = Z.$$

Expression (13) shows that the specific weight of this cone will be a minimum when Z is a minimum. We can then write the condition imposed by the requirement of minimum weight for the anode radiator

$$Z = Z_{\min}. \tag{14}$$

From the obtained conditions (11) and (14) we can solve equation (2) in conjunction with (4)–(6) and obtain the values of k, $\bar{Q}$, and C, corresponding to the lightest anode radiator for the particular case of variation of cross-sectional area of the cone. We denote these values by k_{opt}, Q_{opt}, and C_{opt}. The length of the generatrix of such a cone and its wall thickness are calculated from the formulas

$$L_{\text{opt}} = r_0 k_{\text{opt}} / \sin(\pi/4),$$

$$h_{0\,\text{opt}} = \frac{r_0^2}{4\lambda} \varepsilon\sigma T_0^3 \frac{q^2}{\bar{Q}^2_{\text{opt}} C_{\text{opt}}}, \tag{15}$$

$$h_x = h_0 \varphi/(1 + kt).$$

The results of the numerical solution of Eq. (2) on a digital computer are shown in Fig. 2. As an example, we will take the determination of radiators of optimum weight for the following initial data: anode diameter 50 mm, anode temperature 600° C, heat removed at anode surface 290 kW/m^2, radiator made of copper, and surface emissivity 0.9.

We determine the degree of intensification of heat removal

$$q = 9.8.$$

Using the graphs (Fig. 2) we determine $Z_{\min}$, $\bar{Q}^{-2}_{\text{opt}}$, C_{opt}, and k_{opt}. From formulas (13), and (15) we calculate $\gamma_{a.r.\,\min}$, $h_{0\text{opt}}$ and L_{opt} (see table).

Profiling of the cone wall can greatly reduce the weight of the anode radiator, since replacement of a cone with constant wall thickness by a cone with constant cross-sectional area leads to a reduction of the weight of the radiator by a factor of 1.5, other conditions being equal.

Replacement of a cone with constant cross-sectional area by a cone with linearly decreasing area leads to a reduction of the weight of the radiator by not more than 20%.

The specific weight of the anode radiator, other conditions being equal, is directly proportional to the square of the anode diameter.

NOTATION

T—temperature; F—cross-sectional area; Π—radiating perimeter; x—coordinate along generatrix of cone; L—length of generatrix of cone; λ—thermal conductivity of radiator material; ε—emissivity of surface; σ—Stefan-Boltzmann constant; h—thickness of cone wall; r_0—anode radius; ρ—density of radiator material.

REFERENCES

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27 May 1965